NAG Toolbox for MATLAB

f08kf

1 Purpose

f08kf generates one of the real orthogonal matrices Q or P^{T} which were determined by f08ke when reducing a real matrix to bidiagonal form.

2 Syntax

```
[a, info] = f08kf(vect, k, a, tau, 'm', m, 'n', n)
```

3 Description

f08kf is intended to be used after a call to f08ke, which reduces a real rectangular matrix A to bidiagonal form B by an orthogonal transformation: $A = QBP^{\mathrm{T}}$. f08ke represents the matrices Q and P^{T} as products of elementary reflectors.

This function may be used to generate Q or P^{T} explicitly as square matrices, or in some cases just the leading columns of Q or the leading rows of P^{T}.

The various possibilities are specified by the parameters **vect**, **m**, **n** and **k**. The appropriate values to cover the most likely cases are as follows (assuming that A was an m by n matrix):

1. To form the full m by m matrix Q:

   ```
   [a, info] = f08kf('Q', n, a, tau);
   ```

   (note that the array **a** must have at least m columns).

2. If $m > n$, to form the n leading columns of Q:

   ```
   [a, info] = f08kf('Q', n, a, tau);
   ```

3. To form the full n by n matrix P^{T}:

   ```
   [a, info] = f08kf('P', m, a, tau);
   ```

   (note that the array **a** must have at least n rows).

4. If $m < n$, to form the m leading rows of P^{T}:

   ```
   [a, info] = f08kf('P', m, a, tau);
   ```

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **vect – string**

Indicates whether the orthogonal matrix Q or P^{T} is generated.

vect = 'Q'

Q is generated.

vect = 'P'

P^{T} is generated.

Constraint: **vect** = 'Q' or 'P'.

2: **k – int32 scalar**

If **vect** = 'Q', the number of columns in the original matrix A.

If **vect** = 'P', the number of rows in the original matrix A.

Constraint: $\mathbf{k} \geq 0$.

3: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

Details of the vectors which define the elementary reflectors, as returned by f08ke.

4: **tau(∗) – double array**

Note: the dimension of the array **tau** must be at least $\max(1, \min(\mathbf{m}, \mathbf{k}))$ if **vect** = 'Q' and at least $\max(1, \min(\mathbf{n}, \mathbf{k}))$ if **vect** = 'P'.

Further details of the elementary reflectors, as returned by f08ke in its parameter **tauq** if **vect** = 'Q', or in its parameter **taup** if **vect** = 'P'.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the orthogonal matrix Q or P^{T} to be returned.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the orthogonal matrix Q or P^{T} to be returned.

Constraints:

$\mathbf{n} \geq 0$;
if **vect** = 'Q' and $\mathbf{m} > \mathbf{k}$, $\mathbf{m} \geq \mathbf{n} \geq \mathbf{k}$;
if **vect** = 'Q' and $\mathbf{m} \leq \mathbf{k}$, $\mathbf{m} = \mathbf{n}$;
if **vect** = 'P' and $\mathbf{n} > \mathbf{k}$, $\mathbf{n} \geq \mathbf{m} \geq \mathbf{k}$;
if **vect** = 'P' and $\mathbf{n} \leq \mathbf{k}$, $\mathbf{n} = \mathbf{m}$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The orthogonal matrix Q or P^{T}, or the leading rows or columns thereof, as specified by **vect**, **m** and **n**.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **vect**, 2: **m**, 3: **n**, 4: **k**, 5: **a**, 6: **lda**, 7: **tau**, 8: **work**, 9: **lwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***. A similar statement holds for the computed matrix P^{T}.

8 Further Comments

The total number of floating-point operations for the cases listed in Section 3 are approximately as follows:

1. To form the whole of Q:

   $\frac{4}{3}n\left(3m^2 - 3mn + n^2\right)$ if $m > n$,

   $\frac{4}{3}m^3$ if $m \leq n$;

2. To form the n leading columns of Q when $m > n$:

   $\frac{2}{3}n^2(3m - n)$;

3. To form the whole of P^{T}:

   $\frac{4}{3}n^3$ if $m \geq n$,

   $\frac{4}{3}m\left(3n^2 - 3mn + m^2\right)$ if $m < n$;

4. To form the m leading rows of P^{T} when $m < n$:

   $\frac{2}{3}m^2(3n - m)$.

The complex analogue of this function is f08kt.

9 Example

```
vect = 'P';
k = int32(6);
a = [-0.57, -1.28, -0.39,  0.25;
     -1.93,  1.08, -0.31, -2.14;
```

```
      2.3,   0.24,  0.4,  -0.35;
     -1.93,  0.64, -0.66,  0.08;
      0.15,  0.3,   0.15, -2.13;
     -0.02,  1.03, -1.43,  0.5];
tau = [1.442198259457561;
     1.915944323746201;
     0;
     0];
[a, d, e, tauq, taup, info] = f08ke(a);
[aOut, info] = f08kf(vect, k, a(1:4, :), taup)
```

```
aOut =
    1.0000         0         0         0
         0   -0.4422    0.6732    0.5927
         0   -0.3788   -0.7391    0.5570
         0    0.8130    0.0218    0.5818
info =
           0
```